Math 60 7.1 Simplifying Rational Expressions

Objectives:

1) Evaluate a rational expression at a given value
   a. Substitute the given value for all locations of that variable
   b. Use parentheses if substituting a negative number
   c. Final answer is a number, or "undefined"
2) Determine the values of the variable that make the value of the rational expression undefined
   a. "Undefined" happens when evaluating causes divide by zero.
   b. Only the denominator causes divide by zero.
   c. Set only the denominator equal to zero, and solve
3) Simplify rational expressions
   a. Factor completely
   b. Divide out common factors

Practice:

1) Evaluate $\dfrac{p^2-9}{2p^2+p-10}$ for the following values of p
   a. $p=-1$
   b. $p=0$
   c. $p=2$
   d. $p=-\dfrac{5}{2}$

2) Find the values of the variable that make the expression $\dfrac{3h+2}{h^3+5h^2+4h}$ undefined.

Simplify the rational expression.

3) $\dfrac{4-x^2}{2x^2-x-6}$

4) $\dfrac{ab+3b-ac-3c}{a^2+6a+9}$

5) $\dfrac{x-3}{x^3-27}$

6) $\dfrac{x^3+64}{x^2-4x+16}$

7) $\dfrac{4p^2-20pq+25q^2}{6p^2-7pq-20q^2}$

8) $\dfrac{3x^3+3x^2-36x}{6x^3-6x^2-120x}$

9) $\dfrac{x^3+4x}{x^4-16}$

Math 60 — 7.1 Simplifying Rational Expressions

Objectives: 1) Evaluate a rational expression
2) Determine values of variable that make value of the rational expression undefined.
3) Simplify rational expression
- factor completely
- divide out common factors.

Defn: A rational expression is a quotient of two polynomials $\frac{p}{q}$ where $q \neq 0$.

Generally speaking: a rational expression is a fraction with in variable in the denominator.

① Evaluate $\frac{p^2-9}{2p^2+p-10}$ for a) $p=-1$ b) $p=0$ c) $p=2$ d) $p=-5/2$

Step 1: Substitute (using ( ) for negatives)

Step 2: Use order of operations to calculate.

a) $\frac{p^2-9}{2p^2+p-10} \rightarrow \frac{(-1)^2-9}{2(-1)^2+(-1)-10}$

$= \frac{1-9}{2(1)-1-10}$

$= \frac{-8}{2-1-10}$

$= \frac{-8}{-9}$

$= \boxed{\frac{8}{9}}$

b) $\frac{p^2-9}{2p^2+p-10} \rightarrow \frac{0^2-9}{2(0)^2+0-10}$

$= \frac{-9}{-10}$

$= \boxed{\frac{9}{10}}$

Math SSM 7.1

① cont

c) $\frac{p^2-9}{2p^2+p-10} \rightarrow \frac{2^2-9}{2(2)^2+2-10}$

$= \frac{4-9}{2\cdot 4+2-10}$

$= \frac{-5}{8+2-10}$

$= \frac{-5}{10-10}$

$= \frac{-5}{0}$

$=$ undefined

d) $\frac{p^2-9}{2p^2+p-10} \rightarrow \frac{(-5/2)^2-9}{2(-5/2)^2+(-5/2)-10}$

$= \frac{\frac{25}{4}-9}{2\left(\frac{25}{4}\right)-\frac{5}{2}-10}$

$= \frac{\frac{25}{4}-\frac{9}{1}\cdot\frac{4}{4}}{\frac{50}{4}-\frac{5}{2}\cdot\frac{2}{2}-\frac{10}{1}\cdot\frac{4}{4}}$

$= \frac{\left(\frac{25-36}{4}\right)}{\left(\frac{50-10-40}{4}\right)}$

$= \frac{\frac{-9}{4}}{\left(\frac{0}{4}\right)}$

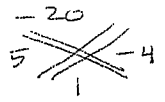
$= \left(\frac{-9}{4}\right) \div \left(\frac{0}{4}\right)$

$= \frac{-9}{4} \div 0$

$=$ undefined

Math 60 SSM 7.1

Review: Solve $2p^2+p-10=0$



$2p^2+5p-4p-10=0$

$p(2p+5)-2(2p+5)=0$

$(2p+5)(p-2)=0$

$2p+5=0$ $\qquad$ $p-2=0$

$2p=-5$ $\qquad$ $\boxed{p=2}$

$\boxed{p=\frac{-5}{2}}$

NOTICE: 1) $2p^2+p-10$ is the denominator of $\frac{p^2-9}{2p^2+p-10}$ in ①.

2) The solutions of $2p^2+p-10=0$ where $-\frac{5}{2}$ and 2.

3) $\frac{-5}{2}$ and 2 were the numbers in ① c) and ① d) that gave undefined results.

4) Undefined means $\div 0$, or denominator $=0$.

5) To find values of variable that make a rational expression undefined, we need denominator $=0$.

② Find the values for which $\frac{3h+2}{h^3+5h^2+4h}$ is undefined.

$h^3+5h^2+4h=0$

$h(h^2+5h+4)=0$

$h(h+4)(h+1)=0$

$\boxed{h=0,-4,-1}$

Math 60 7.1

To simplify a rational expression

Step 0: Write numerator and denominator in standard form. *This is important.*

step 1: Factor everything completely.
This is essential. There is no skipping this step, no matter how close you come.

step 2: Identify common factors in the numerator and divide (cancel) them out.

Step 3: Leave final answer fully factored.
(Don't FOIL or multiply.)

(3) $\dfrac{4-x^2}{2x^2-x-6}$

$= \dfrac{-x^2+4}{2x^2-x-6}$

$= \dfrac{-(x^2-4)}{2x^2-x-6}$

$= \dfrac{-(x+2)(x-2)}{(x-2)(2x+3)}$

$= \boxed{\dfrac{-(x+2)}{(2x+3)}}$

-12, -4, 3, -1

denominator
$2x^2-x-6$
$2x^2-4x+3x-6$
$2x(x-2)+3(x-2)$
$(x-2)(2x+3)$

(4) $\dfrac{ab+3b-ac-3c}{a^2+6a+9}$

$= \dfrac{(a+3)(b-c)}{(a+3)(a+3)}$

$= \boxed{\dfrac{b-c}{a+3}}$

numerator grouping:
$ab+3b-ac-3c$
$b(a+3)-c(a+3)$
$(a+3)(b-c)$

perfect square trinomial for denominator

Math SSM · 7.1

Simplify.

(5) $\dfrac{x-3}{x^3-27}$

diff of cubes
$a^3-b^3=(a-b)(a^2+ab+b^2)$
$x^3-27=(x-3)(x^2+3x+9)$

$= \dfrac{(x-3)}{(x-3)(x^2+3x+9)}$

$= \boxed{\dfrac{1}{x^2+3x+9}}$

(6) $\dfrac{x^3+64}{x^2-4x+16}$

sum of cubes
$a^3+b^3=(a+b)(a^2-ab+b^2)$
$x^3+64=(x+4)(x^2-4x+16)$

Notice! The trinomial is the same as the denominator!

$= \dfrac{(x+4)(x^2-4x+16)}{(x^2-4x+16)}$

$= \boxed{x+4}$

(7) $\dfrac{4p^2-20pq+25q^2}{6p^2-7pq-20q^2}$

numerator
perfect square trinomial
$(2p-5q)(2p-5q)$
$4p^2-10pq-10pq+25q^2$ ✓

denominator
$6p^2-7pq-20q^2$

-120
$-15 \quad 8$
-7

$=6p^2-15pq+8pq-20q^2$

$=3p(2p-5q)+4q(2p-5q)$

$=(2p-5q)(3p+4q)$

$= \boxed{\dfrac{2p-5q}{3p+4q}}$

1, −120
2, −60
3, −40
4, −30
5, −24
6, −20
8, −15

Math 60 7.1

(8) $\dfrac{3x^3+3x^2-36x}{6x^3-6x^2-120x}$

$= \dfrac{3x(x+4)(x-3)}{6x(x+4)(x-5)}$

$= \boxed{\dfrac{(x-3)}{2(x-5)}}$

$\frac{x}{x}=1$

$\frac{3}{6}=\frac{1}{2}$

numerator

$3x^3+3x^2-36x$

$3x(x^2+x-12)$

GCF

-12: 4×-3, $4+(-3)=1$

$3x(x+4)(x-3)$

denom

$6x^3-6x^2-120x$

$6x(x^2-x-20)$

-20: $-5 \times +4$, $-5+4=-1$

$6x(x-5)(x+4)$

(9) $\dfrac{x^3+4x}{x^4-16}$

$= \dfrac{x(x^2+4)}{(x-2)(x+2)(x^2+4)}$

$= \boxed{\dfrac{x}{(x-2)(x+2)}}$

numerator

x^3+4x

$x(x^2+4)$ sum of squares is prime.

denominator

x^4-16 difference of squares

$= (x^2-4)(x^2+4)$ another diff of sq.

$= (x-2)(x+2)(x^2+4)$